MATH 2260 Calculus II
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Name: Answers
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Quiz No. 4

Show all of your work, label your answers clearly, and do not use a calculator.

Problem 1 Determine whether the given series converges or diverges:

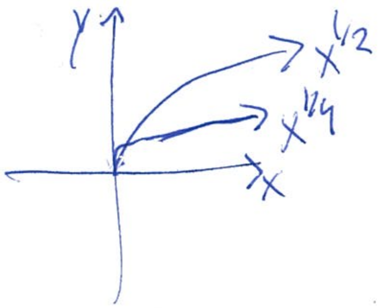
$$\sum_{n=1}^{\infty} \frac{1}{n^3 + n^2}$$

$n^3 \geq 0 \Rightarrow n^3 + n^2 \geq n^2 \Rightarrow \frac{1}{n^3+n^2} \leq \frac{1}{n^2}$

$\sum \frac{1}{n^2}$ conv $\Rightarrow \sum \frac{1}{n^3+n^2}$ conv.

Problem 2 Determine whether the given series converges or diverges:

$$\sum_{n=1}^{\infty} \frac{1}{n^{1/2} + 4n^{1/4}}$$



$\Rightarrow$ for $x \geq 1$, $x^{1/2} \geq x^{1/4}$

$\Rightarrow$ for $n \geq 1$, $\cancel{x}n^{1/2} \geq n^{1/4}$

$\Rightarrow$ $n^{1/2} + 4n^{1/4} \leq n^{1/2} + 4n^{1/2}$

$\Rightarrow$ $n^{1/2} + 4n^{1/4} \leq 5n^{1/2}$

$\Rightarrow$ $\dfrac{1}{n^{1/2} + 4^{1/4}} \geq \dfrac{1}{5n^{1/2}}$

And $\sum_{n=1}^{\infty} \frac{1}{5n^{1/2}}$ diverges (p-series test with $p \leq 1$)

So by comparison $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}+4n^{1/4}}$ diverges as well.

Problem 3 The following is an alternating series. Determine whether or not it converges conditionally, converges absolutely, or diverges:

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^{1/5}}$$

Alternating series test: ① $\frac{1}{n^{1/5}} > 0$

② $n^{1/5}$ increasing $\Rightarrow$ $\frac{1}{n^{1/5}}$ decreasing

③ $\lim_{n\to\infty} \frac{1}{n^{1/5}} = 0$

$\Rightarrow \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^{1/5}}$ converges

However, $\sum_{n=1}^{\infty} \left| (-1)^{n+1} \frac{1}{n^{1/5}} \right| = \sum_{n=1}^{\infty} \frac{1}{n^{1/5}}$ diverges

because $p = 1/5 < 1$, p-series test.

Therefore $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^{1/5}}$ converges conditionally.

Problem 4 Determine whether the given series converges or diverges:

$$\sum_{n=1}^{\infty} \frac{n!}{5^n(n+1)!}$$

Ratio Test:

$$\lim_{n\to\infty} \left(\frac{\frac{(n+1)!}{5^{n+1}(n+2)!}}{\frac{n!}{5^n(n+1)!}} \right) = \lim_{n\to\infty} \frac{(n+1)!(n+1)!\,5^n}{5^{n+1}(n+2)!\,n!}$$

$$= \lim_{n\to\infty} \frac{(n+1)}{(n+2)(5)} = \frac{1}{5} < 1 \quad \text{converges}$$