MATH 2260 Calculus II
Eric Perkerson

Name: Answers
Date: July 2, 2015

Quiz No. 3

Show all of your work, label your answers clearly, and do not use a calculator.

Problem 1 Evaluate the integral

$$\int \frac{6x-5}{x^2-5x+6}dx = \int \frac{6x-5}{(x-2)(x-3)}dx$$

$$\frac{6x-5}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$$

$$6x-5 = A(x-3) + B(x-2)$$

$$x=2 \Rightarrow 12-5 = A(-1) \Rightarrow A = -7$$

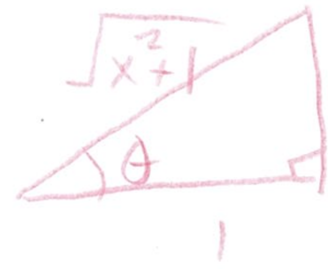
$$x=3 \Rightarrow 18-5 = B(1) \Rightarrow B = 13$$

$$\Rightarrow \int \frac{6x-5}{x^2-5x+6}dx = -7\int \frac{1}{x-2}dx + 13\int \frac{1}{x-3}dx$$

$$= -7\ln|x-2| + 13\ln|x-3| + C$$

Problem 2 Evaluate the integral

$$\int \frac{1}{x^2\sqrt{x^2+1}}dx$$



$\tan(\theta) = x$

$dx = \sec^2(\theta)\, d\theta$

$$\int \frac{1}{x^2\sqrt{x^2+1}}\,dx = \int \frac{1\cdot \sec^2(\theta)}{\tan^2(\theta)\sqrt{\tan^2(\theta)+1}}\,d\theta$$

$$= \int \frac{\sec^2(\theta)}{\tan^2(\theta)\,|\sec(\theta)|}\,d\theta \quad , \quad \text{Assume } \sec(\theta) \geq 0$$

$$= \int \frac{\sec(\theta)}{\tan^2(\theta)}\,d\theta$$

$$= \int \frac{1/\cos(\theta)}{\sin^2(\theta)/\cos^2(\theta)}\,d\theta = \int \frac{\cos(\theta)}{\sin^2(\theta)}\,d\theta$$

$u = \sin(\theta)$, $du = \cos$

$$= \int \frac{1}{u^2}\,du = \frac{-1}{u} + C = \frac{-1}{\sin(\theta)} + C$$

Problem 3 Evaluate the integral

$$\int \cos^5(4t+2)\sin^2(4t+2)dt$$

$$u = 4t+2, \quad du = 4dt$$

$$= \frac{1}{4}\int \cos^5(u)\sin^2(u)\,du$$

$$= \frac{1}{4}\int \left(1-\sin^2(u)\right)^2 \sin^2(u)\cos(u)\,du$$

$$v = \sin(u), \quad dv = \cos(u)du$$

$$= \frac{1}{4}\int \left(1-v^2\right)^2 v^2\,dv$$

$$= \frac{1}{4}\int \left(1-2v^2+v^4\right) v^2\,dv$$

$$= \frac{1}{4}\left(\frac{v^3}{3} - 2\frac{v^5}{5} + \frac{v^7}{7}\right) + C$$

$$= \frac{1}{4}\left(\frac{\sin^3(4t+2)}{3} - 2\frac{\sin^5(4t+2)}{5} + \frac{\sin^7(4t+2)}{7}\right)$$

Problem 4 Evaluate the integral

$$\int \frac{\arctan(x)}{x^2}dx$$

$$= \int \frac{1}{x^2} \arctan(x)\,dx = \int \underbrace{\frac{1}{x^2}}_{f'} \underbrace{\tan^{-1}(x)}_{g}\,dx$$

$$= \left(\frac{-1}{x}\right)\tan^{-1}(x) - \int\left(\frac{-1}{x}\right)\left(\frac{1}{1+x^2}\right)dx$$

$$= \frac{-\tan^{-1}(x)}{x} + \int \frac{1}{x(1+x^2)}dx$$

$$\frac{1}{x(1+x^2)} = \frac{A}{x} + \frac{Bx+C}{1+x^2}$$

$$1 = A(1+x^2) + (Bx+C)(x)$$

$$x = 0 \Rightarrow A = 1$$

$$1 = 1 + x^2 + Bx^2 + Cx$$

$$= (1+B)x^2 + Cx + 1 \Rightarrow 1+B = 0$$

$$C = 0$$

$$\Rightarrow B = -1$$

$$\Rightarrow \int \frac{1}{x(1+x^2)}dx = \int \frac{1}{x}dx + \int \frac{-x}{1+x^2}dx$$

$$u = 1+x^2,\quad du = 2x\,dx$$

$$= \ln|x| - \frac{1}{2}\ln|1+x^2| + C$$

$$\Rightarrow \int \frac{\tan^{-1}(x)}{x^2}dx = \frac{-\tan^{-1}(x)}{x} + \ln|x| - \frac{1}{2}\ln|1+x^2| + C$$