

Practice Midterm

Show all of your work, label your answers clearly, and do not use a calculator.

Problem 1 When the ellipse given by $x^2/3^2 + y^2/2^2 = 1$ is revolved around the horizontal axis we obtain an “ellipsoid of revolution.” Find the volume of the enclosed solid. See the below figure, where $a = 3$ and $b = 2$.

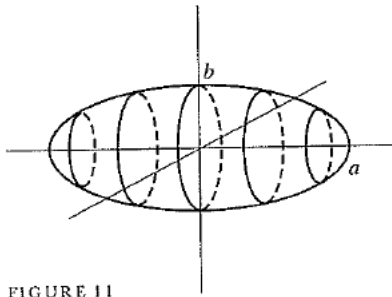


FIGURE 11

Problem 2 Find the volume of the torus (see the figure, where $a = 5$ and $b = 3$) obtained by rotating the circle $(x - 5)^2 + y^2 = 3^2$ around the vertical axis. Note that in the picture, the vertical axis is the y -axis.

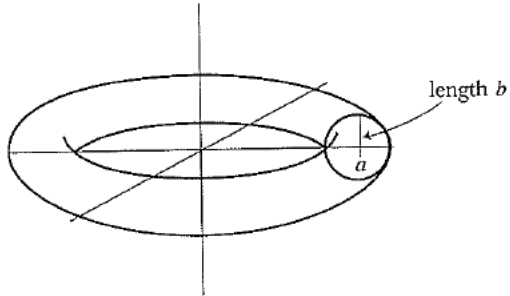


FIGURE 12

Problem 3 Find the length of the curve given by $x = \frac{y^3}{3} + \frac{1}{4y}$ from $y = 1$ to $y = 6$.

Problem 4 Find the area of the surface generated by revolving the curve $x = \frac{e^y + e^{-y}}{2}$ from $0 \leq y \leq \ln(2)$ about the y -axis.

Problem 5 Solve the differential equation given by

$$\frac{dy}{dt} = \frac{5t}{3t(t-1)ye^y}$$

Problem 6 A bathroom scale is compressed $1/12$ in when a 150-lb person stands on it. Assuming that the scale behaves like a spring that obeys Hooke's Law, answer the following:

a *What is the scale's force constant? (Remember the appropriate units!)*

b *How much does someone who compresses the scale $1/8$ in weigh? (Units!)*

c *How much work is done compressing the scale $1/8$ in? (Units!)*

Problem 7 Evaluate the integral

$$\int x \log(x) dx$$

Problem 8 Evaluate the integral

$$\int 6e^{-y} \cos(y) dy$$

Problem 9 Solve the differential equation

$$\int \sec^3(x) dx$$

Problem 10 Evaluate the integral

$$\int \cos^2(x) \sin(x) dx$$

Problem 11 Evaluate the integral

$$\int \frac{1}{\sqrt{9+x^2}} dx$$

Problem 12 Evaluate the integral

$$\int \frac{1}{x^2-1} dx$$

Problem 13 Evaluate the integral

$$\int \frac{4x + 5}{(x + 2)^2} dx$$

Problem 14 Evaluate the integral

$$\int \frac{3x + 1}{(x^2 + 1)^2} dx$$

Problem 15 If the integral

$$\int_1^{\infty} \frac{\cos^2(x)}{x^2} dx$$

converges, give proof by comparison. If it diverges, then prove that by comparison.

Problem 16 If the integral

$$\int_1^{\infty} \frac{1}{1+x^2} dx$$

converges, prove it using any method you wish. If it diverges, then prove that it diverges.