MATH 2250 Calculus I
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Name: Answers
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Test No. 2

Show all of your work, label your answers clearly, and do not use a calculator.

Problem 1 (10 points) Use the technique of linearization, with $f(x) = x^{1/3}$ and $a = 8$ to estimate the value of $\sqrt[3]{7}$.

$$f'(x) = \frac{1}{3}x^{-2/3}$$

$$L(x) = f'(a)(x-a) + f(a)$$

$$= \frac{1}{3}(8)^{-2/3}(x-8) + \sqrt[3]{8}$$

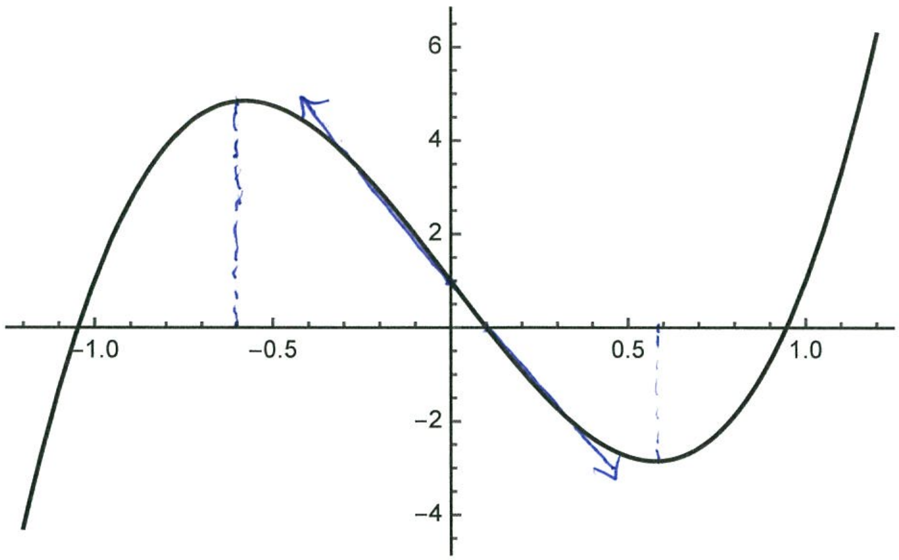
$$= \frac{1}{12}(x-8) + 2$$

$$\Rightarrow L(7) = \frac{1}{12}(-1) + 2$$

$$= \frac{24}{12} - \frac{1}{12}$$

$$= \frac{23}{12}$$

Problem 2 (10 points) For full credit on this problem, make sure that you show your work by showing me how you are using the graph **and** by writing a sentence explaining what you are doing. (You should be drawing a line somewhere on the graph). Given below is the graph of $f(x)$.



a Using the graph, give an estimate of the x-values where $f'(x) = 0$.

$x \approx -0.6, 0.6$

b Estimate $f'(0)$ from the graph.

$$f'(0) \approx \frac{1-0}{0-.1} = \frac{1}{-1/10} = -10$$

Problem 3 (10 points) Find the following derivatives using derivative rules:

a $f(x) = \log(x) - \frac{5}{x^3}$, $f'(x) =$ $\frac{1}{x} - 5(-3)(x^{-4})$

$= \frac{1}{x} + \frac{15}{x^4}$

b $g(x) = \sec(x)(5e^x + x)$, $g'(x) =$ $\sec(x)\tan(x)(5e^x + x) + \sec(x)(5e^x + 1)$

c $h(x) = \pi \frac{\tan(x)}{\cos(x)}$, $h'(x) =$ $\pi\left(\frac{\cos(x)\sec^2(x) + \tan(x)\sin(x)}{\cos^2(x)}\right)$

$= \pi\left(\frac{\sin(x)}{\cos^2(x)}\right)$ $= \pi\left(\frac{\cos^3(x) + 2\sin^2(x)\cos(x)}{\cos^4(x)}\right)$

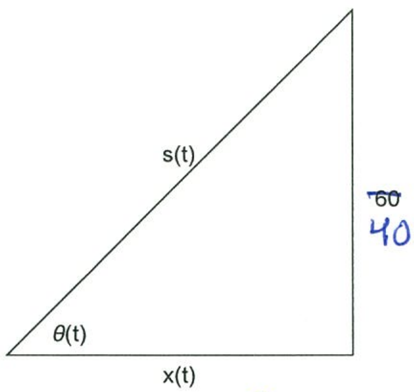
d $y(x) = (\sin(x))^{2015}$, $y'(x) =$

$= 2015(\sin(x))^{2014}\cos(x)$

e $u(x) = \frac{\ln(e^x) + e^{e^x}}{5}$, $u'(x) =$ $\frac{1}{5}(1 + e^{e^x}e^x)$

$= \frac{x + e^{e^x}}{5}$

Problem 4 (20 points) A child is flying a kite, and the wind blows in such a way that the kite is always ~~60~~ 40ft ft above ground (see the picture below). The child lets out the string at a constant rate of 2 ft/sec. Answer the following questions:



$$x^2(t) + 60^2 = s^2(t)$$

$$\cancel{2}x(t)x'(t) = \cancel{2}s(t)s'(t)$$

$$30\, x'(t^*) = 50(2) = 100$$

$$x(t^*) = 30$$

a When there is ~~100~~ 50ft ft of string out, at what rate is the horizontal distance between the child and the kite ($x(t)$ in the picture) changing?

$$x'(t^*) = \frac{10}{3}\ \text{ft/sec}$$

b At this same time, at what rate is the angle $\theta(t)$ changing? (Because radians are a "unit-less" quantity, the unit $\sec^{-1}$ is the same as rad/sec).

$$\theta(t) = \tan^{-1}\left(\frac{40}{x(t)}\right)$$

$$\theta'(t) = \left(\frac{1}{1+\frac{40^2}{x(t)^2}}\right)\left(40(-1)(x(t))^{-2}x'(t)\right)$$

$$\Rightarrow \quad \theta'(t^*) = \frac{-40(10/3)}{x(t^*)^2+40^2}$$

$$\theta'(t^*) = \frac{-40}{30^2+40^2}\left(\frac{10}{3}\right) = \frac{-40}{50^2}\left(\frac{10}{3}\right) = \frac{-4}{250}\left(\frac{10}{3}\right) = \frac{-2}{125}\left(\frac{10}{3}\right)$$

$= -4/$[illegible]

$$\tan(\theta(t)) = \frac{40}{x(t)} = 40(x(t))^{-1}$$

$$\sec^2(\theta(t))\,\theta'(t) = \frac{-40}{x^2(t)}x'(t)$$

$$\left(\frac{s(t^*)}{x(t^*)}\right)\theta'(t) = \frac{-40}{x^2(t^*)}$$

$$\left(\frac{50}{30}\right)\theta'(t) = \frac{-40\left(\frac{10}{3}\right)}{30^2}$$

$$\theta'(t) = \frac{-4\cdot 10}{5\cdot 3\cdot 30}$$

$= -4/\cancel{45}\,75$

Problem 5 (15 points) Given the equation $y^3 + x^3 - 9xy = 0$, do the following:

a Use implicit differentiation to show that $y'(x) = \dfrac{-3x^2 + 9y(x)}{3(y(x))^2 - 9x}$

$$\frac{d}{dx}\left(y^3 + x^3 - 9xy\right) = 0$$

$$3y(x)^2\, y'(x) + 3x^2 - 9\left(y(x) + x y'(x)\right) = 0$$

$$y'(x)\left(3y(x)^2 - 9x\right) = -3x^2 + 9y(x)$$

$$\rightarrow \quad y'(x) = \frac{-3x^2 + 9y(x)}{3y(x)^2 - 9x}$$

b Find the tangent line at the point $(2, 4)$

$$4^3 + 2^3 - 9(2)(4) \stackrel{?}{=} 0$$

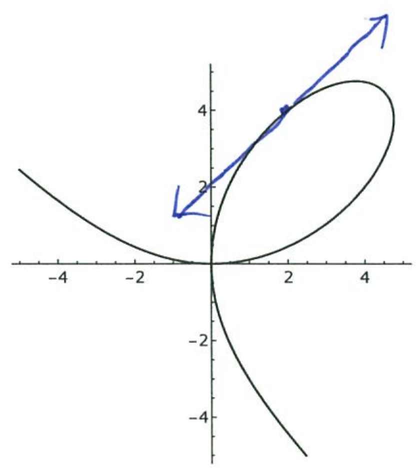
$$(2^2)^3 + 2^3 - 9(2^3) \stackrel{?}{=} 0$$

$$(2^2)^3 - (2^3)(2^3) = 0 \checkmark$$

$$y - 4 = \left(\frac{-3(2^2) + 9(4)}{3(4)^2 - 9(2)}\right)(x - 2$$

slope $= \dfrac{-12 + 36}{48 - 18} = \dfrac{24}{30} = \dfrac{4}{5}$

c Given the curve that satisfies the equation $y^3 + x^3 - 9xy = 0$ below, on the same graph, sketch the tangent line you just found.



Problem 6 (20 points) Astronomers estimate that on a certain exo-planet, a rock thrown upwards from $15m$ above the surface with an initial velocity of $10m/sec$ would have a height above the surface given by the equation $s(t) = -5t^2 + 10t + 15$. (Remember units on your answers).

a Find the rock's velocity and acceleration as function of time t.

$v(t) =$ $-10t + 10$ $\qquad$ $a(t) =$ -10

b How long does it take the rock to reach its highest point?

$0 = -10t + 10 \Rightarrow t = 1$ sec

c How high does the rock go?

$s(1) = -5 + 10 + 15 = 20$ m

d How long is the rock aloft?

$s(t) = 0$

$\Rightarrow -5t^2 + 10t + 15 = 0$

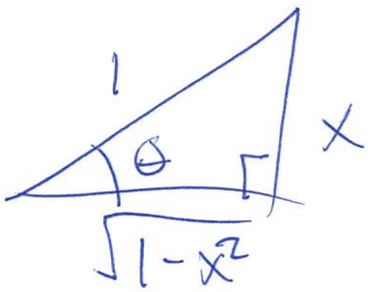
$$\Rightarrow t = \frac{-10 \pm \sqrt{100 - 4(-5)(15)}}{2(-5)}$$

$$= \frac{-10 \pm \sqrt{400}}{-10}$$

$$= \frac{-10 \pm 20}{-10} \therefore 1 \pm 2 = 3$$

Problem 7 (15 points) Using the formula for the derivative of an inverse function, and using the technique of reference triangles to simplify your answer, find $\frac{d}{dx}\arcsin(x)$. (Hint: If you've forgotten the formula for $\frac{d}{dx}\left(f^{-1}(x)\right)$, use implicit differerentiation on the equation $f(f^{-1}(x)) = x$).

$$\frac{\partial}{\partial x}\left(\sin^{-1}(x)\right) = \frac{1}{\cos\left(\sin^{-1}(x)\right)}$$



$$\theta = \sin^{-1}(x)$$

$$\sin(\theta) = x$$

$$\Rightarrow \cos\left(\sin^{-1}(x)\right) =$$

$$\cos(\theta) = \frac{\sqrt{1-x^2}}{1}$$

$$\Rightarrow \frac{\partial}{\partial x}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$$