

## Practice Test No. 4

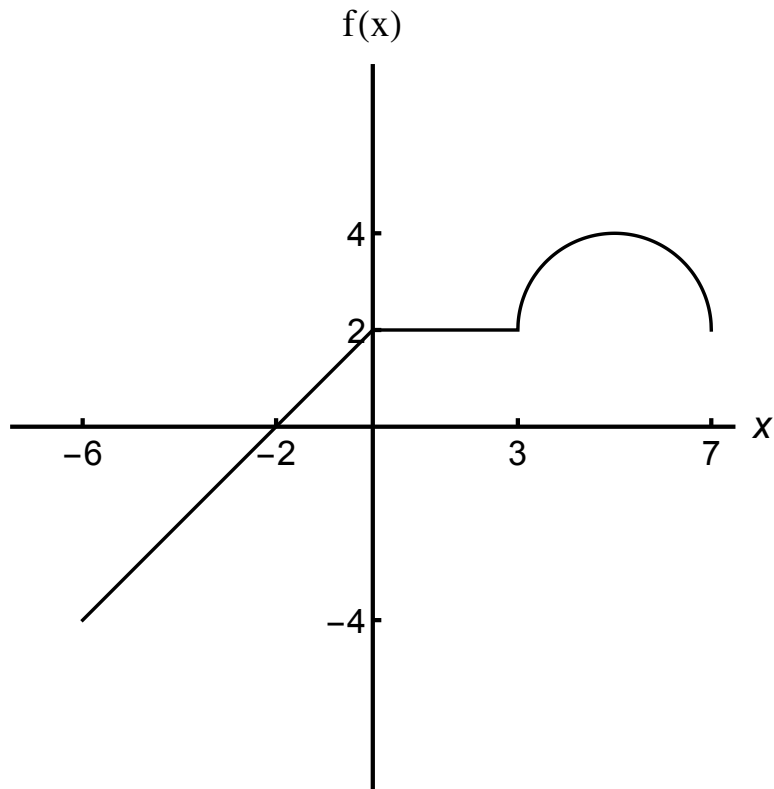
Show all of your work, label your answers clearly, and do not use a calculator.

**Problem 1** (15 points) State both parts of the Fundamental Theorem of Calculus:

**a** Part I

**b** Part II

**Problem 2** (10 points) Find the definite integral  $\int_{-6}^7 f(x)dx$  using the graph of  $f(x)$  given below. Show as much work as you can for partial credit. (The portion of the graph that looks like a semicircle is in fact a semicircle).



**Problem 3** (25 points) Evaluate the following antiderivatives, definite integrals, and average values.

**a**  $\int -2x^6 + \sqrt{x} \, dx$

**b**  $\int \frac{1}{t} + \sin(t) \, dt$

**d**  $\int_0^1 \sec(t) \tan(t) + 1 \, dt$

**e** Find the average value of  $f(x) = \frac{3}{x^2} + 1$  on the interval  $[2, 4]$ .

**Problem 5** (15 points) A small ball is stuck on the end of a spring, and the ball is bouncing up and down on the spring. The vertical acceleration function of the ball is  $a(t) = \sin(t)$ , the velocity at  $t = \pi$  is  $v(\pi) = 1$ , and the position at  $t = \pi$  is  $s(\pi) = 3$ , answer the following questions:

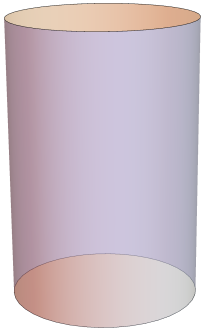
**a** Find the velocity function  $v(t)$  of the particle.

**b** Find the position function  $s(t)$  of the particle.

**c** What is the ball's change in position between  $t = 0$  and  $t = \pi$ ?

**d** How much distance does the ball travel between  $t = 0$  and  $t = \pi$ ?

**Problem 6** (15 points) A container in the shape of a right circular cylinder without a top has a surface area  $5\pi \text{ ft}^2$ . What height and radius will maximize the volume?



**Problem 7** (15 points) For this problem, you will need the formula

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

**a** First use either high school geometry or the Fundamental Theorem of Calculus to compute the definite integral  $\int_0^1 3x + 1 dx$ .

**b** Using the definition and picking  $c_k$  as the right end point of the  $k^{\text{th}}$  interval, write an expression for the Riemann sum in terms of  $n$ , the number of rectangles the interval  $[0, 1]$  is divided up into.

**c** Take the limit of the expression from part b to find the definite integral  $\int_0^1 3x + 1 dx$  using the definition and picking  $c_k$  as the right end point of the  $k^{\text{th}}$  interval.