MATH 2250 Calculus I
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Name: Answers
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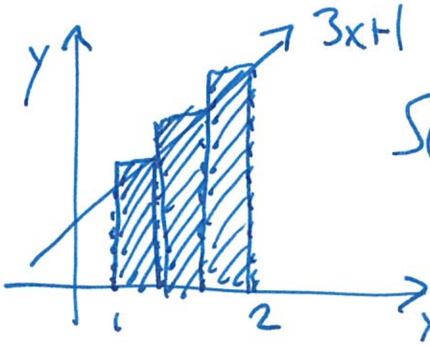
Practice Quiz No. 10

Show all of your work, label your answers clearly, and do not use a calculator.

Problem 1

a *Find the Riemann sum with n rectangles for the function $f(x) = 3x+1$ on the interval $[1,2]$. To simplify, use the formula*

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$



$S(n) = \sum_{k=1}^{n} f(c_k)\Delta x_k$, you can choose c_k to be the left or right endpoint of the k^{th} subinterval (or midpoint).

$b-a=1$, where $[a,b]=[1,2]$

Subintervals: $\left[1+0\left(\frac{1}{n}\right), 1+(1)\left(\frac{1}{n}\right)\right], \left[1+(1)\left(\frac{1}{n}\right), 1+(2)\left(\frac{1}{n}\right)\right], \ldots, \left[1+(n-1)\left(\frac{1}{n}\right), 1+(n)\left(\frac{1}{n}\right)\right]$. Using c_k as right endpoint:

$$\Rightarrow S(n) = \sum_{k=1}^{n}\left(3\left(1+\frac{k}{n}\right)+1\right)\frac{1}{n} = \left(\frac{1}{n}\right)\left(3\left(n+\frac{1}{n}\sum_{k=1}^{n}k\right)+n\right) = \frac{3\left(n+\frac{1}{n}\left(\frac{n(n+1)}{2}\right)\right)+n}{n}$$

$$= \frac{4n+\frac{3}{2}(n+1)}{n}$$

$$= \frac{\frac{11}{2}n+\frac{3}{2}}{n}$$

$$= \frac{11}{2}+\frac{3/2}{n}$$

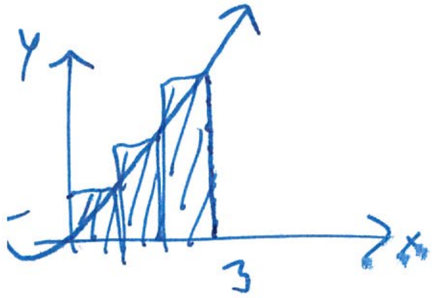
b *Now take the limit of this expression as n goes to infinity.*

$$\lim_{n\to\infty} \frac{11}{2}+\frac{3/2}{n} = \frac{11}{2}$$

Problem 2

a *Find the Riemann sum with n rectangles for the function $f(x) = x^2 + x$ on the interval $[0, 3]$. To simplify, use the formulas*

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \quad \textit{and} \quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$



Choosing c_k to be the right end-point,

$$S(n) = \sum_{k=1}^{n} f(c_k)\Delta x_k = \sum_{k=1}^{n} f\left(k\left(\frac{3}{n}\right)\right)\left(\frac{3}{n}\right)$$

$$= \sum_{k=1}^{n}\left(\left(\frac{3k}{n}\right)^2 + \left(\frac{3k}{n}\right)\right)\left(\frac{3}{n}\right) = \frac{3}{n}\left(\sum_{k=1}^{n}\left(\frac{9}{n^2}k^2 + \frac{3}{n}k\right)\right)$$

$$= \frac{27}{n^3}\left(\sum_{k=1}^{n} k^2\right) + \frac{9}{n^2}\left(\sum_{k=1}^{n} k\right) = \frac{27}{n^3}\left(\frac{\overbrace{n(n+1)}^{n^2+n}(2n+1)}{6}\right) + \frac{9}{n^2}\left(\frac{n(n+1)}{2}\right)$$

$$= \frac{27}{n^3}\left(\frac{2n^3+3n^2+n}{6}\right) + \frac{9}{n^2}\left(\frac{n^2+n}{2}\right) = \frac{27}{3} + \frac{27}{2n} + \frac{27}{6n^2} + \frac{9}{2} + \frac{9}{2n}$$

b *Now take the limit of this expression as n goes to infinity.*

$$\lim_{n\to\infty}\left(\frac{27}{3} + \frac{9}{2} + \frac{27}{2n} + \frac{9}{2n} + \frac{27}{6n^2}\right) = \frac{27}{3} + \frac{9}{2}$$

$$= 9 + \frac{9}{2}$$

$$= \frac{27}{2}$$

You can check this against $\int_0^3 x^2 + x\,dx = \left(\frac{3^3}{3} + \frac{3^2}{2}\right) - \left(\frac{0^3}{3} + \frac{0^2}{2}\right)$

$$= 9 + \frac{9}{2} = \frac{27}{2} \checkmark$$

Problem 3 Using the formulas for the areas of common shapes (e.g. triangles, rectangles, and circles), compute the following definite integrals:

a $\int_{-3}^{5} 2x dx$ $= 16$

b $\int_{-3}^{3} \sqrt{9-x^2} dx$ $= \frac{1}{2}\pi(3^2) = \frac{9\pi}{2}$

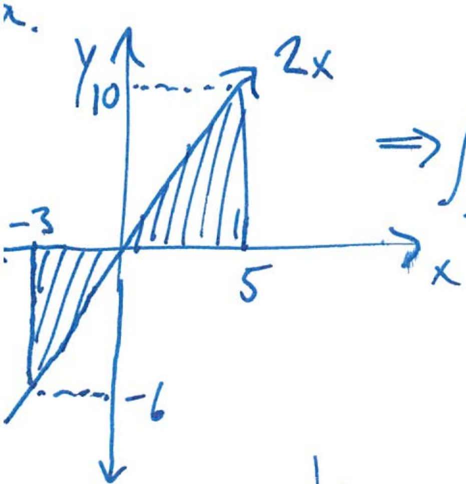
c $\int_{-2}^{1} 4dx$ $= 3(4) = 12$

d $\int_{0}^{5} \sqrt{25-x^2} dx$ $= \frac{25\pi}{4}$

e $\int_{-3}^{5} |x-1| dx$ $= \frac{1}{2}(4)(4) + \frac{1}{2}(4)(5) = 18$

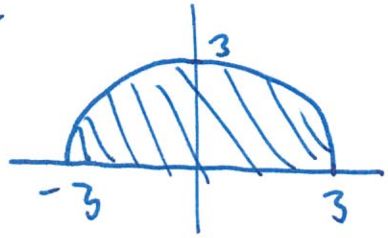
f $\int_{2}^{-5} 3dx$ *(No, it's not a typo.)* $= -\int_{-5}^{2} 3dx = -(7)(3) = -21$

a.

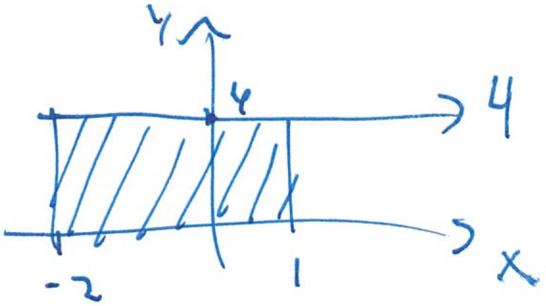


$\Rightarrow \int_{-3}^{5} 2x dx = \frac{1}{2}(5)(10) - \frac{1}{2}(3)(6) = 25 - 9 = 16$

b.



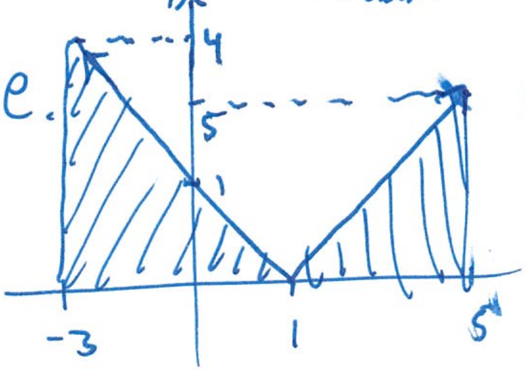
c.



d.

$\Rightarrow \int_{0}^{5} \sqrt{25-x^2} dx = \frac{1}{4}\pi(5)^2$

e.



(Picture not to scale)

f.