MATH 2250 Calculus
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Name:
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Practice Final

Show all of your work, label your answers clearly, and do not use a calculator.

Problem 1 Compute the following limits, showing appropriate justification when necessary.

a $\lim_{x \to -2} \frac{x+2}{x^2+4}$

b $\lim_{x \to 0} \frac{e^x - e^{-x}}{\sin(x)}$

c $\lim_{x \to 0} \frac{x}{\tan(5x)}$

d $\lim_{x \to \infty} \frac{x^2 - 1}{8x^2 + 8x}$

Problem 2 Compute $\frac{dy}{dx}$ for the following functions:

a $y = \dfrac{x^5}{3} - \dfrac{17}{x^2} - \csc^{-1}(x)$

b $y = \dfrac{(2x+5)^3}{e^x + 7}$

c $y = \sin^{-1}(x^2 \cos(x))$

d $y = (x^2 + 1)^{4x}$

e $y = \dfrac{\ln(x)}{\tan(x)}$

Problem 3 Let $f(x) = \sqrt{5x+3}$

a Use the **definition** of the derivative to compute $f'(x)$.

b Find the equation of the tangent line to $f(x)$ at the point $x = 2$.

Problem 4

a Find the average rate of change of the function

$$f(x) = \sqrt{3x + 1}$$

between $x = 1$ and $x = 5$

b Find the point c in the interval $[1, 5]$ where $f'(c)$ equals the average rate computed in part a.

Problem 5 In each part of this problem, x and y satisfy $x^2 - xy + y^2 = 9$.

a Find $\frac{dy}{dx}$ when $(x, y) = (0, 3)$.

b Find all points where the tangent line is horizontal.

Problem 6

a State the Mean Value Theorem.

b Given that a function $f(x)$ satisfies the hypotheses of the Mean Value Theorem for the interval $[-3, 2]$, and we have data $f(2) = -11$ and $f'(x) <= 4$ for all $x \in [-3, 2]$, what is the smallest possible value of $f(-3)$?

Problem 7 A certain function satisfies

$$f(7) = 3$$
$$f'(7) = -5$$
$$f''(7) = 10$$

a Use linear approximation to estimate the value $f(6.9)$.

b Would you expect your answer to overestimate or underestimate the true value? Explain.

Problem 8 Work out the following antiderivatives using any technique that you like.

a $\int \sec(7x+2)\tan(7x+2)dx$

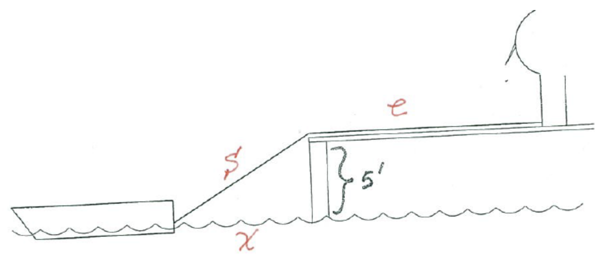
b $\int \frac{x}{(x^2+4)^{17}}dx$

c $\int \sqrt{7x-2}dx$

d $\int \frac{x}{x^2+1}dx$

Problem 9 Consider a trapezoid sketched in the coordinate plane. Two vertices of the trapezoid are located at $(-2, 0)$ and $(2, 0)$, and the other two lie on the semicircle given by $y = \sqrt{4 - x^2}$. What is the maximum possible area of such a trapezoid? (Note: the area of any trapezoid with bases b_1 and b_2 and height h is $(1/2)h(b_1 + b_2)$).

Problem 10 A 20 foot rope is tied to the back of a boat. It is then stretched over the edge of a 5 foot high dock and tied to the ankle of an elephant who happens to be standing on the dock (see figure below). If the boat moves away from the dock at 10 ft/sec, how fast is the elephant dragged when the boat is 12 feet from the base of the dock?



Problem 11 Suppose we know the following facts about an unknown function $f(x)$ with domain all real numbers:

$$\lim_{x\to-\infty} = -\infty \quad \lim_{x\to\infty} = \infty$$
$$f(0) = 7 \quad f'(0) = 0$$
$$f(2) = 1 \quad f'(2) = 0$$
$$f'(x) > 0 \text{ for } x < 0, \quad x > 2$$
$$f'(x) < 0 \text{ for } 0 < x < 2$$
$$f''(x) < 0 \text{ for } x < 1$$
$$f''(x) > 0 \text{ for } x > 1$$

Find:

a The interval(s) where f is increasing.

b The x-coordinate(s) where f has a local maximum.

c The x-coordinate(s) where f has a local minimum.

d The interval(s) where f is concave up.

e Now graph the function below.

Problem 12 You are building the frame for a Christmas tree. The frame of the tree consists of a circular floor and a vertical central pole extending from the floor to the apex of the cone. The conical cavity will be filled with 9 cubic yards of flame-resistant stuffing. The floor costs \$2 per square yard, and the pole costs \$6 per yard. What are the dimensions of the least expensive frame possible? (Remember, $V = (1/3)\pi r^2 h$ for a cone).

Problem 13 State both parts of the Fundamental Theorem of Calculus:

a Part I

b Part II

Problem 14

a Use part one of the Fundamental Theorem of Calculus to compute:

$$\frac{d}{dx}\int_1^x \sqrt{t^2+\sin t}\,dt$$

b Use part two of the Fundamental Theorem of Calculus to compute:

$$\int_{\pi/6}^{\pi/4} \csc^2(t)\,dt$$

Problem 15

a Evaluate $\int_1^3 \frac{1}{x} dx$.

b Draw a graph of $y = 1/x$ below, and on that graph, sketch in four right-endpoint rectangles to approximate the value you computed in part a above.

c Now compute your area estimate using the four right-endpoint rectangles.

Problem 16 Find the area of the region enclosed by the two curves

$$y = 2x, \qquad y = x^3$$

Note that this region has pieces lying in both the first and third quadrants.

Problem 17 Dr. Teddy is wearing a bungee cord and a rocket pack. He jumps off a bridge at time $t = 0$. We measure Dr. Teddy's position function in feet ABOVE the bridge, t minutes after jumping. Dr. Teddy's acceleration upward after t minutes is given by the function:

$$a(t) = 6\sin(3t) \text{ ft/min}^2$$

a If the initial velocity is $v(0) = -1$ ft/min, find Dr. Teddy's velocity function $v(t)$ in ft/min.

b Given that $s(0) = 0$, find Dr. Teddy's position function $s(t)$ in ft.

c When is the first time that Dr. Teddy changes direction?

Problem 18 The graph in the figure below shows the rate of change of the depth of a small pond in cm/day taken over a period of 9 days. The interval from 0 to 1 represents Day 1, the interval from 1 to 2 represents Day 2, and so on. At the beginning of Day 1, the pond starts with depth 200 cm.

a What is the longest run of days during which the level of the pond decreased every day?

b At the end of which day is the depth of the pond greatest?

c Is the depth of the pond greater than or less than 200 cm at the end of the nine day period?