MATH 1113 Precalculus
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Name: Solutions
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Quiz No. 5

DUE WEDNESDAY JULY 19th IN CLASS.
Show all of your work, label your answers clearly, and you MAY use a calculator. You MAY use your class notes and the textbook. You CANNOT work with or discuss the quiz with anyone else. You CANNOT use any material other than your notes and the textbook.

10/10

Problem 1 Find an angle coterminal to θ on the interval $[0, 2\pi]$

a $\frac{94\pi}{3}$

$$\frac{94\pi}{3} - 15(2\pi) = \boxed{\frac{4\pi}{3}} \in [0, 2\pi]$$

b $\frac{-17\pi}{8}$

$$\frac{-17\pi}{8} + 2(2\pi) = \boxed{\frac{15\pi}{8}} \in [0, 2\pi]$$

15/15

Problem 2 Convert the given angles

a $\frac{4\pi}{5}$ radians to degrees.

$$\frac{4\pi}{5}\left(\frac{180^\circ}{\pi}\right) = 144^\circ$$

b 732° to radians

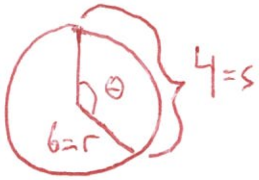
$$732^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{61\pi}{15} = 12.7758$$

c 32° to revolutions (yes, revolutions, not radians)

$$32^\circ\left(\frac{1 \text{ rev}}{360^\circ}\right) = \frac{4}{45} \text{ rev} = 0.0\overline{8} \text{ rev}$$

5/5

Problem 3 You are given that the length of an arc on a circle is 4, and that the radius of the circle is 6. What angle (in radians, of course) centered at the center of the circle gives you this arc?



$$s = r\theta = 6\theta$$

$$\Rightarrow \quad 4 = 6\theta$$

$$\Rightarrow \quad \theta = \frac{4}{6} = \frac{2}{3}$$

In degrees, $\frac{2}{3} = \frac{2}{3}\left(\frac{180^\circ}{\pi}\right) =$

5/5

Problem 4 Given that the area of a circular sector is 13π and that the angle at the center of the circle that gives you this sector is $\pi/7$, what is the radius of the circle?

$$13\pi = \left(\frac{\theta}{2\pi}\right)(\pi r^2)$$

$$\Rightarrow \quad 13\pi = \left(\frac{\pi/7}{2\pi}\right)(\pi r^2)$$

$$13\pi = \frac{\pi}{14} r^2$$

$$13(14) = r^2$$

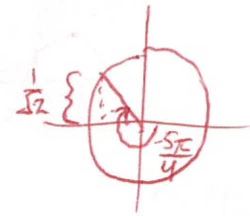
$$182 = r^2$$

$$\Rightarrow \quad r = \sqrt{182},$$ because r must be positive.

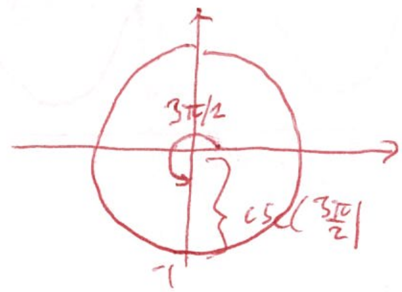
Problem 5 Evaluate the following and show your answer on the unit circle. (Decimal answer will receive zero credit.)

30/30

a $\sin(\frac{-5\pi}{4})$ $= \frac{1}{\sqrt{2}}$



b $\csc(\frac{3\pi}{2})$ $= \frac{1}{\sin(\frac{3\pi}{2})} = \frac{1}{-1} = -1$



c Given that $\tan(\frac{-\pi}{8}) = 1 - \sqrt{2}$, find $\cot(\frac{7\pi}{8})$

$\tan(-\frac{\pi}{8}) = \tan(\frac{7\pi}{8}) = 1-\sqrt{2}$, so $\cot(\frac{7\pi}{8}) = \frac{1}{1-\sqrt{2}}$

d Given that $\sin(\theta) = \frac{-5}{13}$, find $\cos(\theta)$ using one of the Pythagorean identites.

$\cos^2(\theta) + \sin^2(\theta) = 1$

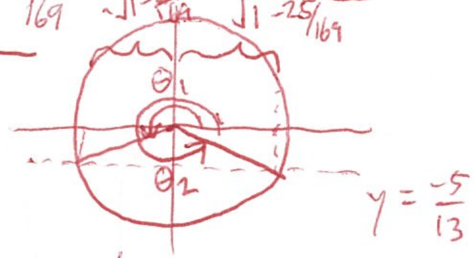
$\cos^2(\theta) + (\frac{-5}{13})^2 = 1$

$\cos^2(\theta) = 1 - \frac{25}{169}$

$\cos(\theta) = \pm\sqrt{1 - \frac{25}{169}}$

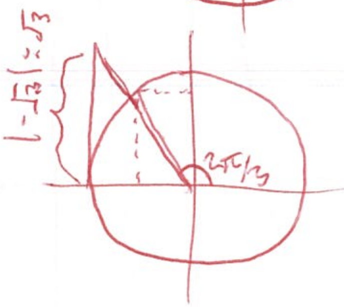
$= \pm\sqrt{\frac{144}{169}}$

$= \pm\frac{12}{13}$

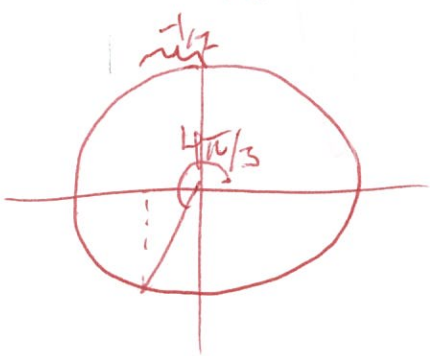


e $\tan(\frac{2\pi}{3})$

$= \frac{\sin(2\pi/3)}{\cos(2\pi/3)} = \frac{\sqrt{3}/2}{-1/2} = -\sqrt{3}$



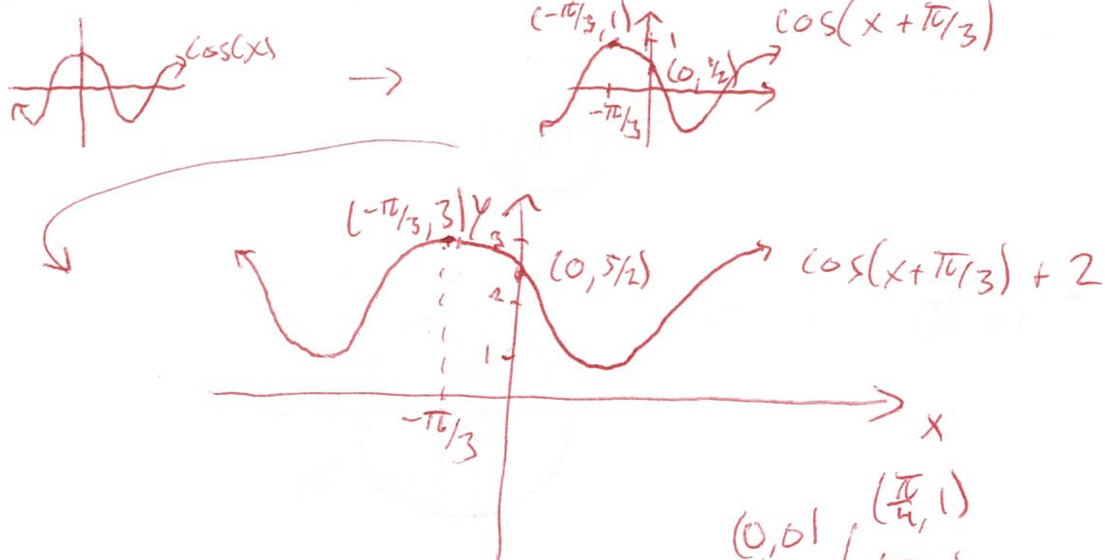
f $\cos(\frac{4\pi}{3})$ $= -1/2$



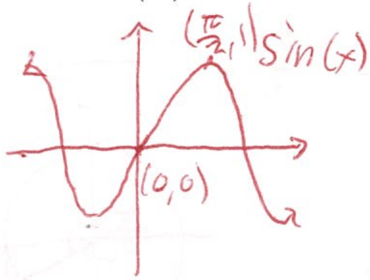
15/15

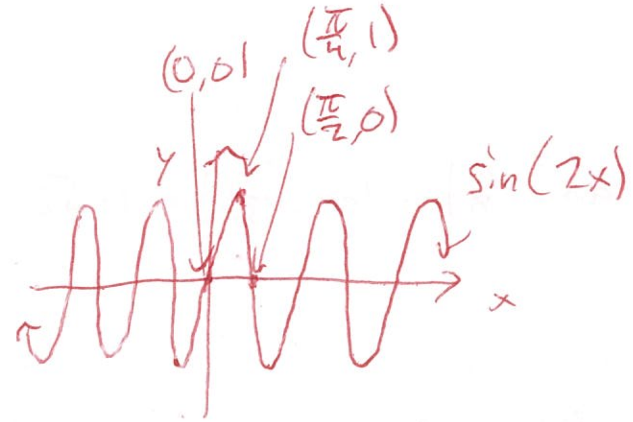
Problem 6 Graph the functions:

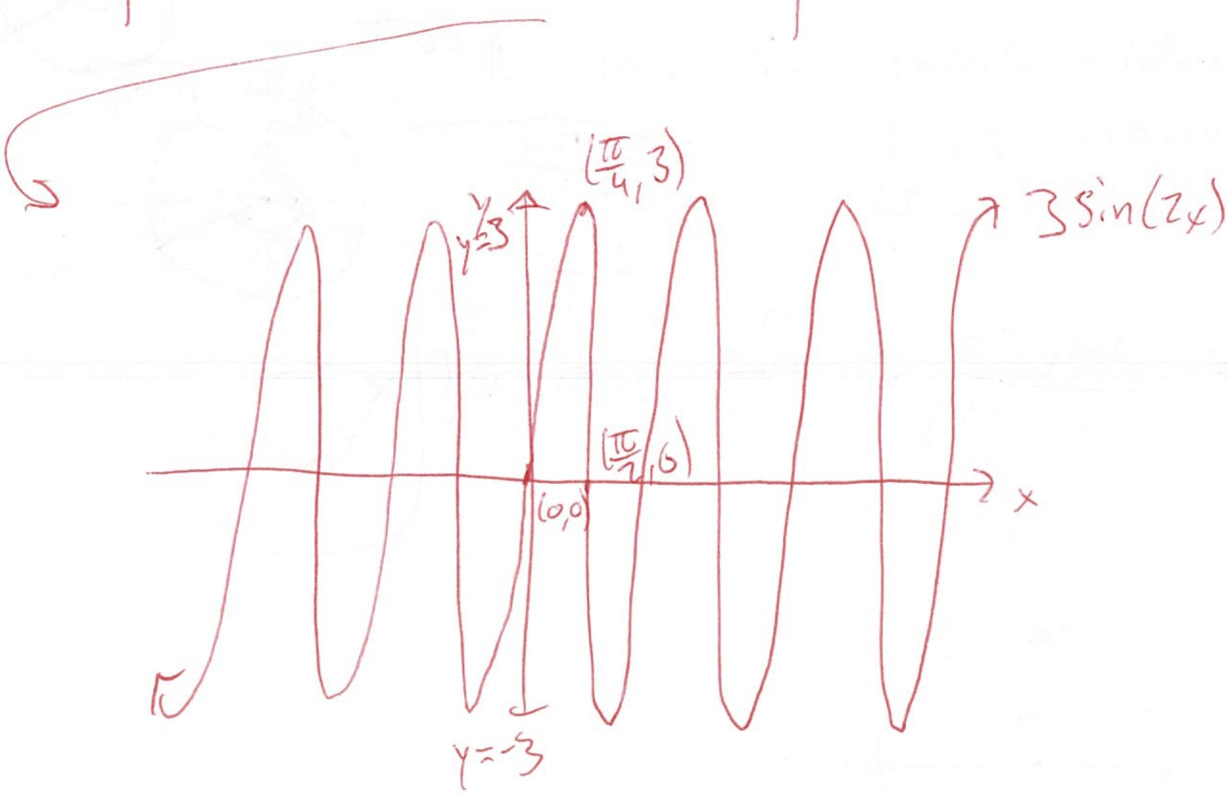
a $\cos(x + \pi/3) + 2$



b $3\sin(2x)$







20/20

Problem 7

a A circular sector can be folded up into a right-circular cone. If your circular sector has angle θ, and the cirlce it is part of has radius R, what is the circumference of the base of the right-circular cone it can be folded into? (Try this with a piece of paper if you do not understand what is going on).

This length becomes the circumference of the base of the cone,

so $S = R\theta$ is the circumference of the base of the cone.

b What is the radius of the base of the cone, now that you know the circumference?

$2\pi r$ is the circumference of the base of the cone, so $2\pi r = R\theta \Rightarrow r = \frac{R\theta}{2\pi}$ is the radius of the base of the cone.

c What is the height of the cone?

$$\Rightarrow h^2 + r^2 = R^2$$

$$h^2 + \left(\frac{R\theta}{2\pi}\right)^2 = R^2$$

$$h^2 = R^2 - \left(\frac{R\theta}{2\pi}\right)^2.$$

$$\Rightarrow h = \sqrt{R^2 - \left(\frac{R\theta}{2\pi}\right)^2}$$

↑ positive.

d What is the surface area of the cone? Include the area of the side of the cone and the area of the base of the cone.

$$\text{Area} = \underbrace{\left(\tfrac{1}{2}\theta R^2\right)}_{\text{Area of side}} + \underbrace{\left(\pi\left(\frac{R\theta}{2\pi}\right)^2\right)}_{\text{Area of base.}}$$