MATH 1113 Precalculus
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Name: Solutions
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Quiz No. 4

Show all of your work, label your answers clearly, and do not use a calculator.

Problem 1 Using the change of base formula, write

a $\log_2(15)$ in base e

$$\log_2(15) = \frac{\log_e(15)}{\log_e(2)} = \frac{\ln(15)}{\ln(2)}$$

b $\log_5(30)$ in base 10

$$\log_5(30) = \frac{\log_{10}(30)}{\log_{10}(5)} = \frac{\log(30)}{\log(5)}$$

Problem 2 If a couple has \$80,000 in a retirement account, how long will it take the money to grow to \$1,000,000 if it grows by 6% interest compounded continuously?

$$A(t) = Pe^{rt}$$

$$1000000 = 80000e^{.06t}$$

$$\frac{1000000}{80000} = e^{.06t}$$

$$\ln\left(\frac{50}{4}\right) = .06t$$

$$\frac{\ln(50/4)}{.06} = t$$

$$\frac{\ln(25/2)}{.06} = t$$

Problem 3 Solve the following equations:

a $\log_2(x) = 4 - \log_2(x-6)$

$\log_2(x) + \log_2(x-6) = 4$

$\log_2(x(x-6)) = 4$

$x(x-6) = 2^4$

$\to x^2 - 6x - 16 = 0$

$x = \dfrac{6 \pm \sqrt{36 - 4(1)(-16)}}{2}$

$= \dfrac{6 \pm 10}{2} = 8, \cancel{-2}$

Doesn't make sense in original eqn.

b $e^{2x} - 9e^x - 22 = 0$

$e^x = \dfrac{9 \pm \sqrt{81 - 4(1)(-22)}}{2}$

$= \dfrac{9 \pm 13}{2} = 11, \cancel{-2} \Rightarrow x = \ln(11)$

c $801 = 23^x + 6$

$795 = 23^x$

$\log_{23}(795) = x$

d $\log_3(n-5) + \log_3(n+3) = 2$

$\log_3((n-5)(n+3)) = 2$

$(n-5)(n+3) = 3^2 = 9$

$n^2 - 2n - 15 - 9 = 0$

$n^2 - 2n - 24 = 0$

$\to (n-6)(n+4) = 0$

$\Rightarrow n = 6, \cancel{-4}$

Doesn't make sense in original equation

Problem 4 The isotope of plutonium ^{238}Pu is used to make thermoelectric power sources for spacecraft. Suppose that a space probe was launched in 2012 with 2.0 kg of ^{238}Pu.

a If the half-life of ^{238}Pu is 87.7 years, write a function of the form $Q(t) = Q_0 e^{-kt}$ to model the quantity of ^{238}Pu left after t years.

$Q_0 = 2$, (half-life) $= 87.7$, so $Q(87.7) = 1$

$\Rightarrow 1 = 2e^{-k(87.7)} \Rightarrow \ln(\frac{1}{2}) = -k(87.7) \Rightarrow k = \frac{-\ln(1/2)}{87.7}$

$\Rightarrow Q(t) = 2e^{-\left(\frac{-\ln(1/2)}{87.7}\right)t} = 2e^{\frac{\ln(1/2)}{87.7}t}$

b If 1.6 kg of ^{238}Pu is required to power the spacecraft's data transmitter, for how long after launch would scientists be able to receive data?

Set $1.6 = Q(t)$

$1.6 = 2e^{\left(\frac{\ln(1/2)}{87.7}\right)t}$

$\ln\left(\frac{1.6}{2}\right) = \frac{\ln(1/2)}{87.7}t$

$\frac{87.7\ln(1.6/2)}{\ln(1/2)} = t$

Problem 5 After a new product is launched the cumulative sales $S(t)$ (in thousands of dollars) t weeks after launch is given by

$$S(t) = \frac{72}{1 + 9e^{-0.36t}}$$

a Determine the cumulative amount of sales 3 weeks after launch.

$$S(3) = \frac{72}{1+9e^{-.36(3)}} \quad \text{(in thousands)}$$

b Determine the amount of time required for the cumulative sales to reach \$70,000.

Set $70 = S(t)$

$$70 = \frac{72}{1+9e^{-.36t}}$$

$$(1+9e^{-.36t})70 = 72$$

$$9e^{-.36t} = \frac{72}{70} - 1$$

$$e^{-.36t} = \frac{1}{9}\left(\frac{72}{70} - 1\right)$$

$$-.36t = \ln\left(\frac{1}{9}\left(\frac{72}{70} - 1\right)\right)$$

$$t = \frac{1}{-.36}\ln\left(\frac{1}{9}\left(\frac{72}{70} - 1\right)\right)$$

c What is the limiting value in sales?

72 (thousand)