MATH 1113 Precalculus
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Name: Solutions
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Practice Test No. 2

Show all of your work, label your answers clearly, and do not use a calculator.

Problem 1

a Describe in words what it means for a function to be one-to-one (Your answer shouldn't just be "it passes the horizontal line test.")

A function $f(x)$ is one-to-one if for every y-value there is at most one x value such that $f(x) = y$.

b Why do non-one-to-one functions not have inverse functions?

Because if there are two different x-values x_1 and x_2 where $f(x_1) = f(x_2) = y$, then $f^{-1}(y)$ would have to be x_1 and x_2, so f^{-1} wouldn't be a function.

c What is the inverse function of $f(x) = 5x^3 + 2$?

$$y = 5x^3 + 2$$

$$\left(\frac{y-2}{5}\right)^{1/3} = x \Rightarrow f^{-1}(x) = \left(\frac{x-2}{5}\right)^{1/3}$$

d What is the inverse function of $f(x) = 3e^{5x}$?

$$y = 3e^{5x}$$

$$\frac{y}{3} = e^{5x}$$

$$\ln(y/3) = 5x$$

$$x = \frac{\ln(y/3)}{5}$$

$$\Rightarrow f^{-1}(x) = \frac{\ln(x/3)}{5}$$

Problem 2

a If you deposit \$300 in a savings account that pays 3% annual interest, compounded monthly, how much money would you have after 4 years?

$$A(4) = 300\left(1 + \frac{.03}{12}\right)^{12(4)}$$

b If you deposit \$300 in a savings account that pays 3% annual interest, compounded continuously, how much money would you have after 4 years?

$$A(4) = 300e^{.03(4)}$$

c If \$300 in a savings account compounded continuously grows to \$500 after 18 years, what was the annual interest rate?

$$500 = 300e^{r(18)}$$

$$\frac{500}{300} = e^{18r}$$

$$\ln(5/3) = 18r$$

$$r = \frac{\ln(5/3)}{18}$$

Problem 3

a One day you discover an unidentified radioactive isotope in your lab. If you start with 4 grams of the isotope, and then 6 years later you run a test and find that only 1.3 grams of the material is remaining, what must the half-life of your isotope be?

$$A(0) = 4 = A_0 e^{k(0)} = A_0$$

$$A(6) = 1.3 = A_0 e^{k6} = 4e^{k6} \Rightarrow \frac{\ln\left(\frac{1.3}{4}\right)}{6} = k$$

So set $A(t) = 4/2 = 2 \Rightarrow 2 = 4e^{\left(\frac{\ln(1.3/4)}{6}\right)t}$

$$\Rightarrow \frac{2}{4} = \frac{1}{2} = e^{\frac{\ln(1.3/4)}{6}t} \Rightarrow \ln(1/2)\left(\frac{6}{\ln(1.3/4)}\right)$$ is the half-life.

b What function models the amount of radioactive isotope as a function of time in years?

$$A(t) = 4e^{\left(\ln(1.3/4)/6\right)t}$$

c How many years until you would only have 0.2 grams of the isotope?

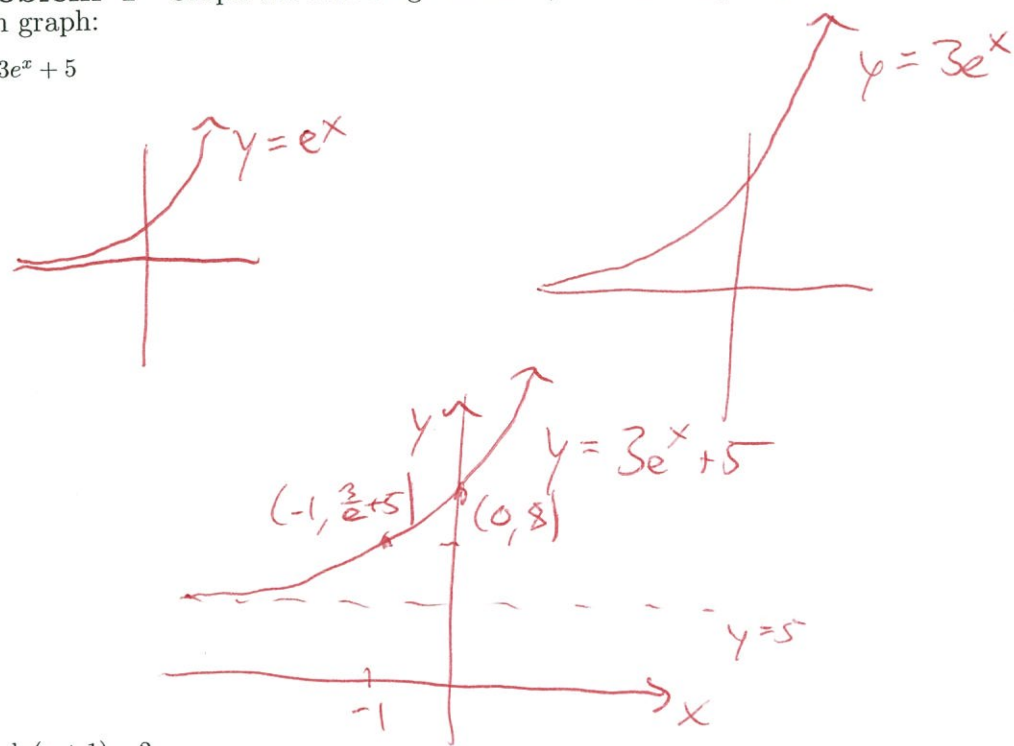
$$.2 = 4e^{\frac{\ln(1.3/4)}{6}t}$$

$$\ln\left(\frac{.2}{4}\right) = \frac{\ln(1.3/4)}{6}t$$

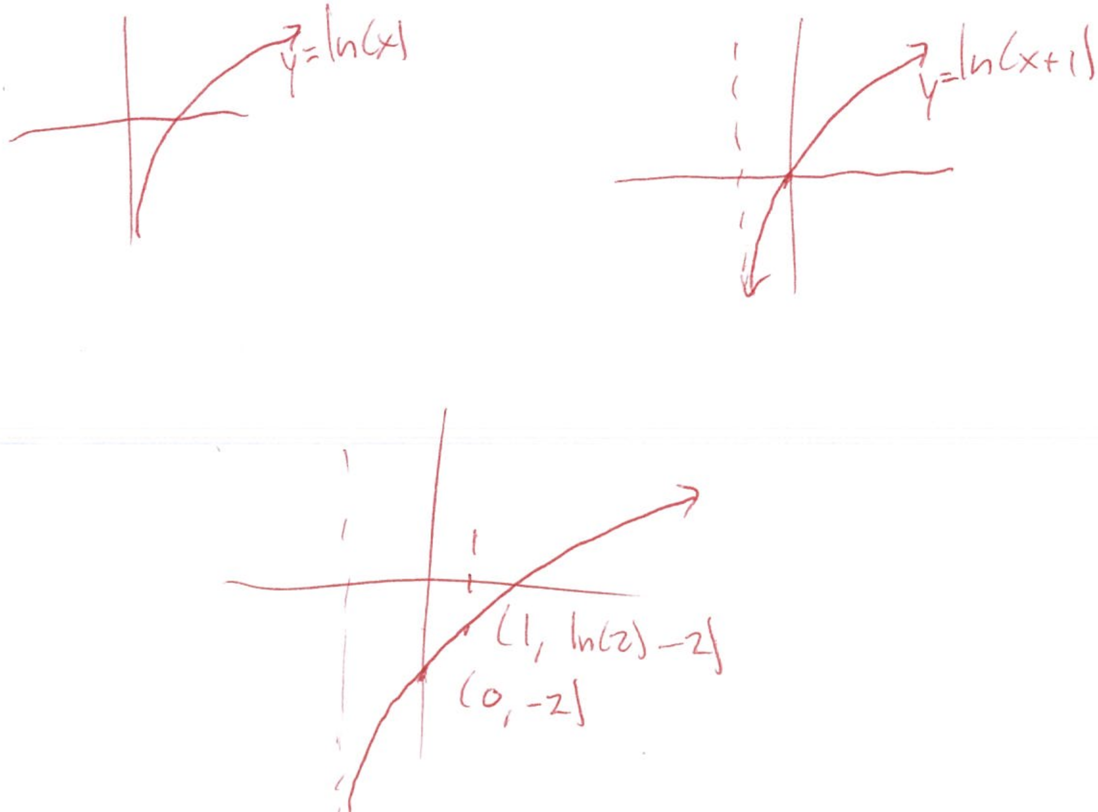
$$t = \frac{6\ln(.2/4)}{\ln(1.3/4)}$$

Problem 4 Graph the following functions, remembering to plot at least two points on each graph:

a $3e^x + 5$



b $\ln(x + 1) - 2$



Problem 5 Solve the following equations:

a $4\log_3(2t-7) = 8$

$$\log_3(2t-7) = 2$$
$$2t-7 = 3^2$$
$$2t = 9+7$$
$$t = 8$$

b $\ln(x-5) = \ln(x+4) - \ln(x)$

$$\ln(x-5) = \ln((x+4)/x)$$
$$x-5 = \frac{x+4}{x}$$
$$x^2-5x = x+4$$
$$x^2-6x-4 = 0$$

$$x = \frac{6 \pm \sqrt{36-4(-4)}}{2}$$
$$x = \frac{6 \pm \sqrt{52}}{2}$$
$$x = \frac{6+\sqrt{52}}{2}, \quad \cancel{\frac{6-\sqrt{52}}{2}}$$

Doesn't make sense in original equation. Not defined

c $3e^{2x} - 2e^{x} - 25 = 0$

$$3(e^x)^2 - 2e^x - 25 = 0$$

Let $e^x = u$

$$3u^2 - 2u - 25 = 0$$
$$u = \frac{2 \pm \sqrt{4-4(3)(-25)}}{6}$$
$$e^x = \frac{2 \pm \sqrt{304}}{6}$$
$$x = \ln\left(\frac{2+\sqrt{304}}{6}\right), \quad \cancel{\ln\left(\frac{2-\sqrt{304}}{6}\right)}$$

Problem 6 The population of Canada $P(t)$ (in millions) since January 1, 1990, can be approximated by

$$P(t) = \frac{55.1}{1 + 9e^{-0.02515t}}$$

where t is the number of years since January 1, 1990.

a Evaluate $P(0)$ and interpret its meaning in the context of this problem.

$$P(0) = \frac{55.1}{1+9e^0} = \frac{55.1}{10} = 5.51 \text{ (million)}$$

is the pop. of Canada on Jan 1, 1990.

b Use the function to approximate the Canadian population on January 1, 2015.

$2015 - 1990 = 25$, so

$$P(25) = \frac{55.1}{1 + 9e^{-.02515(25)}} \text{ million}$$

c From the model, when would the Canadian population be 45 million?

$$45 = \frac{55.1}{1+9e^{-.02515t}}$$

$$(1+9e^{-.02515t})45 = 55.1$$

$$\frac{\frac{55.1}{45} - 1}{9} = e^{-0.02515t}$$

$$t = \frac{\ln\left(\frac{55.1/45 - 1}{9}\right)}{-.02515}$$

d Determine the limiting value of $P(t)$.

55.1 (million)

Problem 7 A bank account will be opened, and the interest rate is 2.7% compounded quarterly. How long will it take the money to triple?

$$A(t) = P\left(1 + \frac{.027}{4}\right)^{4t}$$

$$3P = P\left(1 + \frac{.027}{4}\right)^{4t}$$

$$3 = \left(1 + \frac{.027}{4}\right)^{4t}$$

$$\log(3) = 4t \log\left(1 + \frac{.027}{4}\right)$$

$$\frac{\log(3)}{4 \log\left(1 + \frac{.027}{4}\right)} = t$$