MATH 1113 Precalculus
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Name: Solutions
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Practice Test No. 1

Show all of your work, label your answers clearly, and do not use a calculator.

Problem 1

a Describe in words what it means for a relation to be a function

A relation is a function if every x-value is related to at most one y-value.

b Does the relation $y^2 = x$ define y as a function of x?

No, because x is related to $\sqrt{y}$ and to $-\sqrt{y}$.

Problem 2

(25 points) Find the equations of the following lines:

a The line parallel to the y-axis going through the point $(-3, 2)$.

$$x = -3$$

b The line perpendicular to $y = -2x + 2$ going through the point $(\frac{-3}{4}, \frac{5}{2})$.

$$m = \frac{-1}{-2} = \frac{1}{2}$$

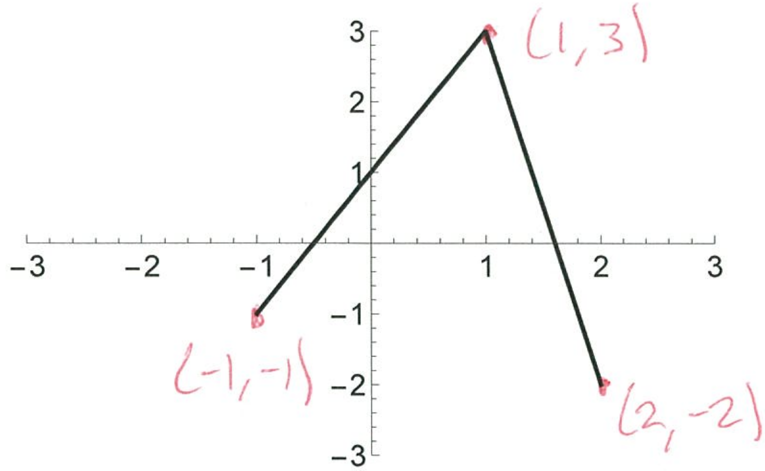
$$\Rightarrow \quad y - \frac{5}{2} = \frac{1}{2}\left(x - \left(\frac{-3}{4}\right)\right)$$

c The line going through the two points $(-1, \frac{7}{5})$ and $(-3, -15)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-15 - 7/5}{-3 - (-1)}$$

$$\Rightarrow \quad y - (-15) = \left(\frac{-15 - 7/5}{-3 - (-1)}\right)(x - (-1))$$

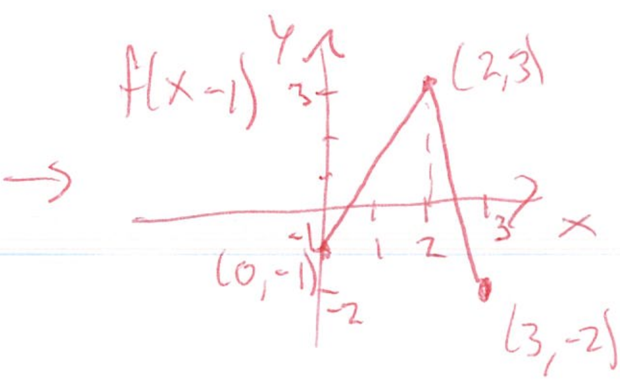
Problem 3 (25 points) Given the graph of $f(x)$ below:

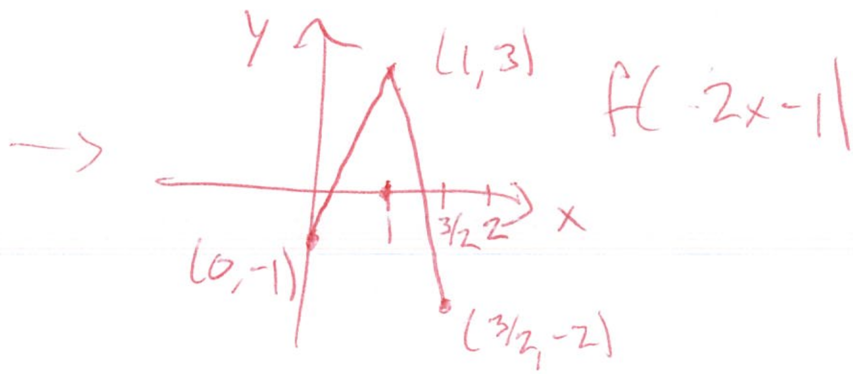


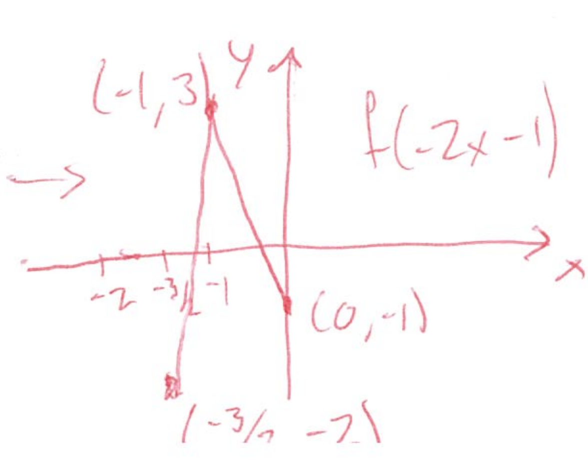
a Describe in words all of the graph transformations needed to transform $f(x)$ into $g(x) = 3f(-2x-1)$.

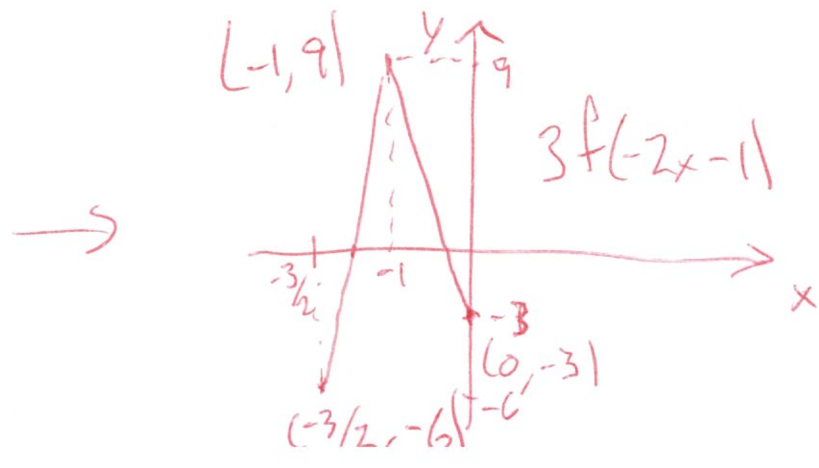
Horizontal translation by 1 to the right.
Horizontal shrinkage by doing $\frac{1}{2}$ times x-coor.
Reflection across y-axis
Vertical stretch by doing 3 times y-coor.

b Graph the function $g(x) = 3f(-2x-1)$.



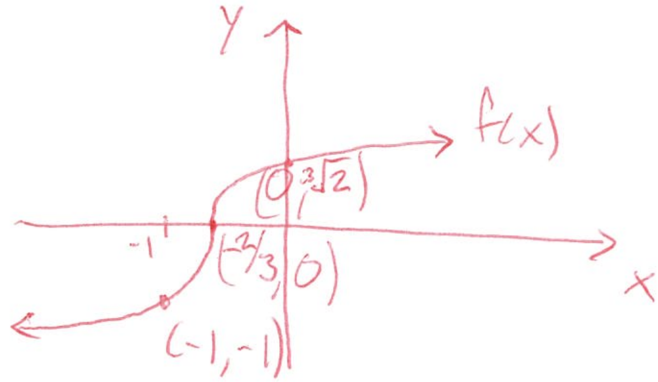




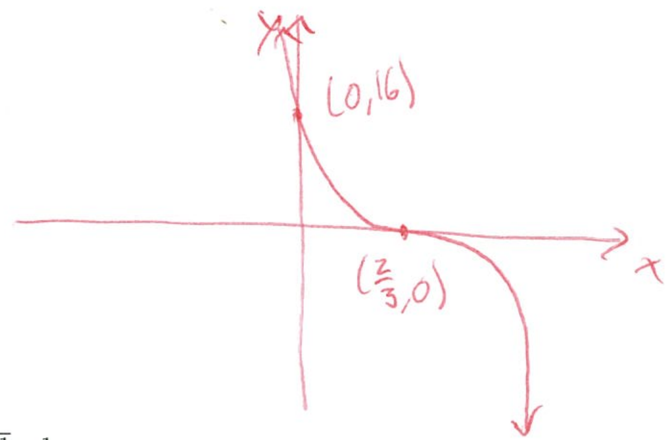


Problem 4 (25 points) Graph each of the following functions:

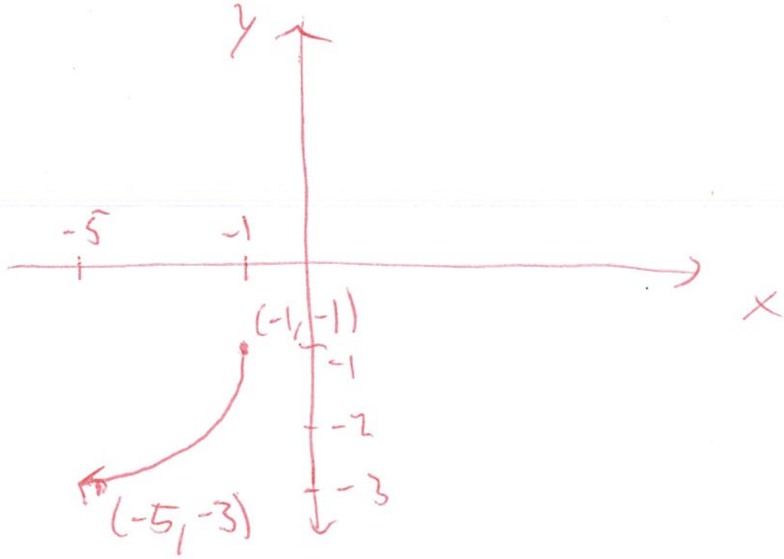
a $f(x) = (3x+2)^{1/3}$



b $g(x) = 2(-3x+2)^3$



c $r(t) = -\sqrt{-x-1} - 1$



Problem 5 For each of the following functions, say whether the function is even, odd, or neither.

a $f(x) = x^2 - 1$

Check: $f(-x) = (-x)^2 - 1 = x^2 - 1 = f(x)$

So f is even

b $s(m) = \frac{m^2}{m^4 - 3m^2 + 1}$

Check: $s(-m) = \frac{(-m)^2}{(-m)^4 - 3(-m)^2 + 1} = \frac{m^2}{m^4 - 3m^2 + 1}$

$= s(m)$ so s is even

c $g(t) = \frac{t}{t^2+1}$

Check: $g(-t) = \frac{(-t)}{(-t)^2 + 1} = \frac{-t}{t^2+1} = -\left(\frac{t}{t^2+1}\right)$

$= -g(t)$ so g is odd

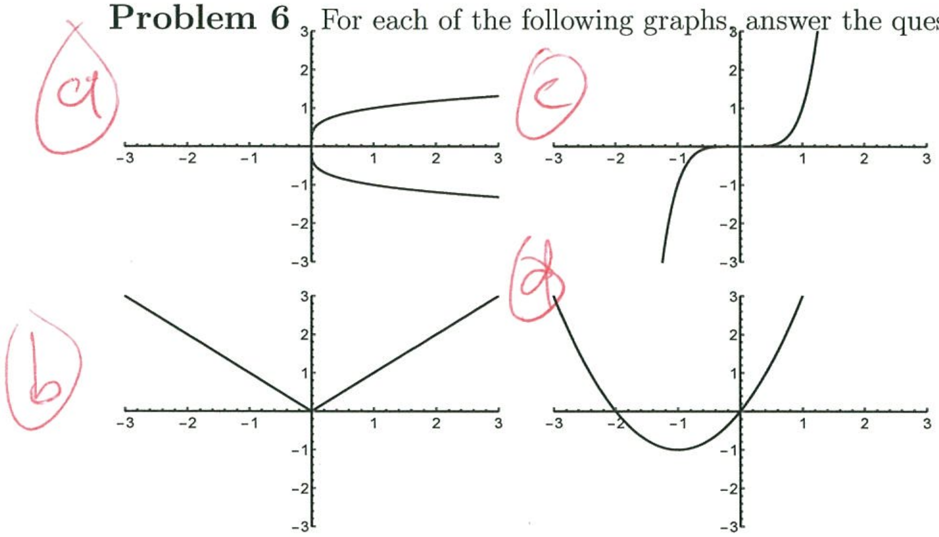
d $y(x) = |x - 3|$

Check: $y(-x) = |(-x) - 3| = |-(x+3)|$

$= |x+3| \neq y(x)$ and $\neq -y(x)$

So $y(x)$ is neither.

Problem 6 For each of the following graphs, answer the questions below.



a Can the graph be represented as the graph of a function of x?

a. No
b. Yes
c. Yes
d. Yes

b Is the graph symmetric across the origin?

a. No
b. Yes
c. No
d. No

c Is the graph symmetrix across the x-axis?

a. Yes
b. No
c. No
d. No

d Is the graph symmetrix across the y-axis?

a. No
b. No
c. Yes
d. No

Problem 7 A company founded in the year 0 started with 20 employees and hired 60 new employees every year for the next 30 years. For the next 6 years, they did not hire any new employees. For the next 10 years, they laid off 10 employees per year.

a Write an expression for the function $m(t)$ that represents the number of employees the company had in each year t.

$$m(t) = \begin{cases} 60t + 20, & 0 \leq t \leq 30 \\ 60(30) + 20 = 1820, & 30 \leq t \leq 36 \\ -10(x-36) + 1820, & 36 \leq t \leq 46 \end{cases}$$

Has $m = -10$, goes through $(36, 1820)$

b What is the domain of $m(t)$?

$\text{dom}(m) = [0, 46]$

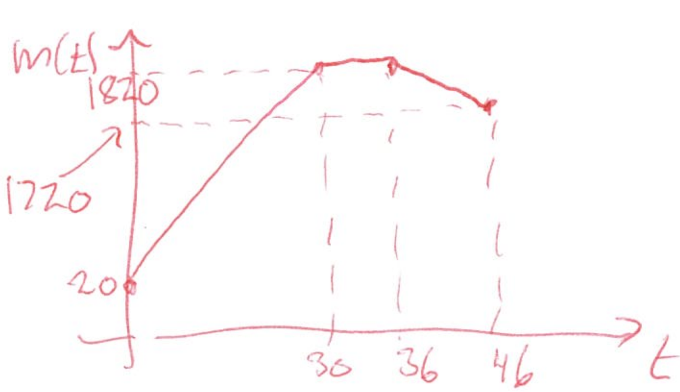
c What is the range of $m(t)$?

$\text{range}(m) = [20, 1820]$

d What is the maximum of $m(t)$?

$y = 1820$

e Sketch a graph of the function $m(t)$.



Problem 8 Given the function $f(x) = -3x^2 + 5x + 15$.

a Determine the average rate of change of f on the interval $[0, 3]$

$a = 0, \quad b = 3$

$$(\text{Avg. rate of change}) = \frac{f(b) - f(a)}{b - a} = \frac{-3(3)^2 + 5(3) + 15 - f(0)}{3 - 0}$$

$$= \frac{-27 + 15}{3} = \frac{-12}{3} = -4$$

b Determine the average rate of change of f on the interval $[1, 2]$

$a = 1, \quad b = 2$

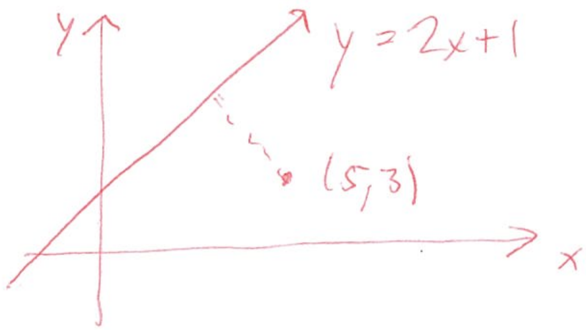
$$(\text{Avg. rate of change}) = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{[-3(2)^2 + 5(2) + 15] - [-3(1)^2 + 5(1) + 15]}{2 - 1}$$

$$= -12 + 10 + 15 + 3 - 5 - 15$$

$$= -4$$

Problem 9 Find the point on the line $y = 2x + 1$ closest to the point $(5, 3)$.



Any point on the line $y=2x+1$ can be written $(x, 2x+1)$, and the distance from $(x, 2x+1)$ to the point $(5,3)$ is $d = \sqrt{(x-5)^2 + ((2x+1)-3)^2}$

$$\Rightarrow d^2 = (x-5)^2 + (2x-2)^2$$
$$= x^2 - 10x + 25 + 4x^2 - 8x + 4$$
$$= 5x^2 - 18x + 29$$
$$= 5\left(x^2 - \tfrac{18}{5}x\right) + 29$$
$$= 5\left(x - \cancel{\tfrac{18}{5}} + \left(\tfrac{9}{5}\right)^2 - \left(\tfrac{9}{5}\right)^2\right) + 29$$
$$= 5\left(\left(x - \tfrac{9}{5}\right)^2 - \underbrace{\left(\tfrac{9}{5}\right)^2\Big) + 29}_{\text{We don't need the y-coor.}}$$

$\frac{-18}{5} = -2x_0 \Rightarrow x_0 = \frac{18}{10} = \frac{9}{5}$

$h = \frac{9}{5}$ is the x-coordinate of the vertex, hence the argmin of d^2.

$x = \frac{9}{5}$ is also the argmin of d (Squaring doesn't change where the minimum is.)

So $\left(\frac{9}{5}, 2\left(\frac{9}{5}\right) + 1\right)$ is the closest point on $y=2x+1$ to $(5,3)$.